

## Lecture 10: Empirical Covering Number Bounds (October 5)

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For infinite function classes, earlier we had uniform one-sided deviation bounds based on bracketing numbers. As discussed earlier, bracketing numbers do not give good bounds for classifications. So we are going to replace them with  $L_1$  uniform covering numbers. Before we state the theorem, we will need another lemma apart from the symmetrization lemma:

**Lemma 10.1.** *Let*

1.  $P \in \mathcal{M}_1(\mathcal{Z})$ ,  $(Z, Z') \sim P^{\otimes n}$ ,
2.  $\psi : \mathcal{F} \times \mathcal{Z}^n \rightarrow \mathbb{R}$ ,  $\psi' : \mathcal{F} \times \mathcal{Z}^n \rightarrow \mathbb{R}$ ,  $\psi : \mathcal{F} \rightarrow \mathbb{R}$ ,
3.  $0 < \delta_1, \delta_2 < 1$ ,  $\varepsilon > 0$ .

*Assume*

- (U) w.p.  $1 - \delta_1$ ,  $\forall f$ ,  $\psi'(f, Z') \leq \psi(f, Z)$ .
- (NU)  $\forall f$ , w.p.  $1 - \delta_2$ ,  $\psi(f) \leq \psi'(f, Z') + \varepsilon$ .

*Then w.p.  $1 - \delta_1 - \delta_2$ ,  $\forall f$*

$$\psi(f) \leq \psi(f, Z) + \varepsilon.$$

*Proof.*

□