Lecture 10: Empirical Covering Number Bounds (October 5)

Lecturer: Csaba Szepesvári *Scribes: Kushagra Chandak*

For infinite function classes, earlier we had uniform one-sided deviation bounds based on bracketing numbers. As discussed earlier, bracketing numbers do not give good bounds for classifications. So we are going to replace them with L_1 uniform covering numbers. Before we state the theorem, we will need another lemma apart from the symmetrization lemma:

Lemma 10.1. *Let*

1. $P \in \mathcal{M}_1(\mathcal{Z})$, $(Z, Z') \sim P^{\otimes n}$,
2. $\psi : \mathcal{F} \times \mathcal{Z}^n \to \mathbb{R}$, $\psi' : \mathcal{F} \times \mathcal{Z}^n \to \mathbb{R}$, $\psi : \mathcal{F} \to \mathbb{R}$,
3. $0 < \delta_1, \delta_2 < 1$, $\varepsilon > 0$.

Assume

- *(U) w.p.* $1 - \delta_1$, $\forall f$, $\psi'(f, Z') \le \psi(f, Z)$.
- *(NU)* $\forall f$, *w.p.* $1 - \delta_2$, $\psi(f) \le \psi'(f, Z') + \varepsilon$.

Then w.p. $1 - \delta_1 - \delta_2$, $\forall f$

$$\psi(f) \le \psi(f, Z) + \varepsilon\,.$$

Proof. □